WIKIPEDIA

Hexadecimal

In mathematics and computing, **hexadecimal** (also **base 16**, or **hex**) is a positional system that represents numbers using a base of 16. Unlike the common way of representing numbers with ten symbols, it uses sixteen distinct symbols, most often the symbols "0"–"9" to represent values zero to nine, and "A"–"F" (or alternatively "a"–"f") to represent values ten to fifteen.

Hexadecimal numerals are widely used by computer system designers and programmers, as they provide a human-friendly representation of binary-coded values. Each hexadecimal digit represents four binary digits, also known as a nibble, which is half a byte. For example, a single byte can have values ranging from 00000000 to 11111111 in binary form, which can be conveniently represented as 00 to FF in hexadecimal.

In mathematics, a subscript is typically used to specify the base, also known as the radix. For example, the decimal value 10,995 would be expressed in hexadecimal as $2AF3_{16}$. In programming, a number of notations are used to support hexadecimal representation, usually involving a prefix or suffix. The prefix `0x` is used in C and related languages, which would denote this value by `0x2AF3`.

Hexadecimal is used in the transfer encoding **Base16**, in which each byte of the plaintext is broken into two 4-bit values and represented by two hexadecimal digits.

Contents

Representation

Written representation

Using 0–9 and A–F

In contexts where the base is not clear, hexadecimal numbers can be ambiguous and confused with numbers expressed in other bases. There are several conventions for expressing values unambiguously. A numerical subscript (itself written in decimal) can give the base explicitly: 159_{10} is decimal 159; 159_{16} is hexadecimal 159, which is equal to 345_{10}. Some authors prefer a text subscript, such as 159_{decimal} and 159_{hex}, or 159_{d} and 159_{h}.

0$_{\text{hex}}$ = 0$_{\text{dec}}$ = 0$_{\text{oct}}$	0	0	0	0
1$_{\text{hex}}$ = 1$_{\text{dec}}$ = 1$_{\text{oct}}$	0	0	0	1
2$_{\text{hex}}$ = 2$_{\text{dec}}$ = 2$_{\text{oct}}$	0	0	1	0
3$_{\text{hex}}$ = 3$_{\text{dec}}$ = 3$_{\text{oct}}$	0	0	1	1
4$_{\text{hex}}$ = 4$_{\text{dec}}$ = 4$_{\text{oct}}$	0	1	0	0
5$_{\text{hex}}$ = 5$_{\text{dec}}$ = 5$_{\text{oct}}$	0	1	0	1
6$_{\text{hex}}$ = 6$_{\text{dec}}$ = 6$_{\text{oct}}$	0	1	1	0
7$_{\text{hex}}$ = 7$_{\text{dec}}$ = 7$_{\text{oct}}$	0	1	1	1
8$_{\text{hex}}$ = 8$_{\text{dec}}$ = 10$_{\text{oct}}$	1	0	0	0
9$_{\text{hex}}$ = 9$_{\text{dec}}$ = 11$_{\text{oct}}$	1	0	0	1
A$_{\text{hex}}$ = 10$_{\text{dec}}$ = 12$_{\text{oct}}$	1	0	1	0
B$_{\text{hex}}$ = 11$_{\text{dec}}$ = 13$_{\text{oct}}$	1	0	1	1
C$_{\text{hex}}$ = 12$_{\text{dec}}$ = 14$_{\text{oct}}$	1	1	0	0
D$_{\text{hex}}$ = 13$_{\text{dec}}$ = 15$_{\text{oct}}$	1	1	0	1
E$_{\text{hex}}$ = 14$_{\text{dec}}$ = 16$_{\text{oct}}$	1	1	1	0
F$_{\text{hex}}$ = 15$_{\text{dec}}$ = 17$_{\text{oct}}$	1	1	1	1

In linear text systems, such as those used in most computer programming environments, a variety of methods have arisen:

- In URIs (including URLs), character codes are written as hexadecimal pairs prefixed with `%`: `http://www.example.com /name%20with%20spaces` where `%20` is the code for the space (blank) character, ASCII code point 20 in hex, 32 in decimal.
- In XML and XHTML, characters can be expressed as hexadecimal numeric character references using the notation `&#x`*code*`;`, where the *x* denotes that *code* is a hex code point (of 1- to 6-digits) assigned to the character in the Unicode standard. Thus `&#x2019;` represents the right single quotation mark (’), Unicode code point number 2019 in hex, 8217 (thus `&#8217;` in decimal).[1]
- In the Unicode standard, a character value is represented with `U+` followed by the hex value, e.g. `U+20AC` is the Euro sign (€).
- Color references in HTML, CSS and X Window can be expressed with six hexadecimal digits (two each for the red, green and blue components, in that order) prefixed with #: white, for example, is represented as #FFFFFF.[2] CSS also allows 3-hexdigit abbreviations with one hexdigit per component: #FA3 abbreviates #FFAA33 (a golden orange:).
- Unix (and related) shells, AT&T assembly language and likewise the C programming language (and its syntactic descendants such as C++, C#, D, Java, JavaScript, Python and Windows PowerShell) use the prefix `0x` for numeric constants represented in hex: `0x5A3`. Character and string constants may express character codes in hexadecimal with the prefix `\x` followed by two hex digits: `'\x1B'` represents the Esc control character; `"\x1B[0m\x1B[25;1H"` is a string containing 11 characters (plus a trailing NUL to mark the end of the string) with two embedded Esc characters.[3] To output an integer as hexadecimal with the printf function family, the format conversion code `%X` or `%x` is used.
- In MIME (e-mail extensions) quoted-printable encoding, characters that cannot be represented as literal ASCII characters are represented by their codes as two hexadecimal digits (in ASCII) prefixed by an *equal to* sign =, as in `Espa=F1a` to send "España" (Spain). (Hexadecimal F1, equal to decimal 241, is the code number for the lower case n with tilde in the ISO/IEC 8859-1 character set.[4])
- In Intel-derived assembly languages and Modula-2,[5] hexadecimal is denoted with a suffixed `H` or `h`: `FFh` or `05A3H`. Some implementations require a leading zero when the first hexadecimal digit character is not a decimal digit, so one would write `0FFh` instead of `FFh`
- Other assembly languages (6502, Motorola), Pascal, Delphi, some versions of BASIC (Commodore), GameMaker Language, Godot and Forth use `$` as a prefix: `$5A3`.
- Some assembly languages (Microchip) use the notation `H'ABCD'` (for ABCD$_{16}$). Similarly, Fortran 95 uses `Z'ABCD'`.
- Ada and VHDL enclose hexadecimal numerals in based "numeric quotes": `16#5A3#`. For bit vector constants VHDL uses the notation `x"5A3"`.[6]
- Verilog represents hexadecimal constants in the form `8'hFF`, where 8 is the number of bits in the value and FF is the hexadecimal constant.
- The Smalltalk language uses the prefix `16r`: `16r5A3`
- PostScript and the Bourne shell and its derivatives denote hex with prefix `16#`: `16#5A3`. For PostScript, binary data (such as image pixels) can be expressed as unprefixed consecutive hexadecimal pairs:

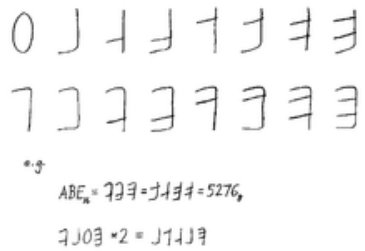
`AA213FD51B3801043FBC...`

- Common Lisp uses the prefixes #x and #16r. Setting the variables *read-base*[7] and *print-base*[8] to 16 can also be used to switch the reader and printer of a Common Lisp system to Hexadecimal number representation for reading and printing numbers. Thus Hexadecimal numbers can be represented without the #x or #16r prefix code, when the input or output base has been changed to 16.
- MSX BASIC,[9] QuickBASIC, FreeBASIC and Visual Basic prefix hexadecimal numbers with &H: `&H5A3`
- BBC BASIC and Locomotive BASIC use & for hex.[10]
- TI-89 and 92 series uses a `0h` prefix: `0h5A3`
- ALGOL 68 uses the prefix `16r` to denote hexadecimal numbers: `16r5a3`. Binary, quaternary (base-4) and octal numbers can be specified similarly.
- The most common format for hexadecimal on IBM mainframes (zSeries) and midrange computers (IBM System i) running the traditional OS's (zOS, zVSE, zVM, TPF, IBM i) is `X'5A3'`, and is used in Assembler, PL/I, COBOL, JCL, scripts, commands and other places. This format was common on other (and now obsolete) IBM systems as well. Occasionally quotation marks were used instead of apostrophes.
- Donald Knuth introduced the use of a particular typeface to represent a particular radix in his book *The TeXbook*.[11] Hexadecimal representations are written there in a typewriter typeface: `5A3`
- Any IPv6 address can be written as eight groups of four hexadecimal digits (sometimes called hextets), where each group is separated by a colon (`:`). This, for example, is a valid IPv6 address: `2001:0db8:85a3:0000:0000:8a2e:0370:7334`; this can be abbreviated as `2001:db8:85a3::8a2e:370:7334`. By contrast, IPv4 addresses are usually written in decimal.
- Globally unique identifiers are written as thirty-two hexadecimal digits, often in unequal hyphen-separated groupings, for example `{3F2504E0-4F89-41D3-9A0C-0305E82C3301}`.

There is no universal convention to use lowercase or uppercase for the letter digits, and each is prevalent or preferred in particular environments by community standards or convention.

History of written representations

The use of the letters *A* through *F* to represent the digits above 9 was not universal in the early history of computers.



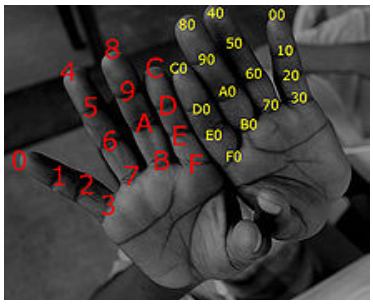
Bruce Alan Martin's hexadecimal notation proposal[12]

- During the 1950s, some installations favored using the digits 0 through 5 with an overline to denote the values 10–15 as $\overline{0}$, $\overline{1}$, $\overline{2}$, $\overline{3}$, $\overline{4}$ and $\overline{5}$.
- The SWAC (1950)[13] and Bendix G-15 (1956)[14][13] computers used the lowercase letters *u*, *v*, *w*, *x*, *y* and *z* for the values 10 to 15.
- The ILLIAC I (1952) computer used the uppercase letters *K*, *S*, *N*, *J*, *F* and *L* for the values 10 to 15.[15][13]
- The Librascope LGP-30 (1956) used the letters *F*, *G*, *J*, *K*, *Q* and *W* for the values 10 to 15.[16][13]
- The Honeywell Datamatic D-1000 (1957) used the lowercase letters *b*, *c*, *d*, *e*, *f*, and *g* whereas the Elbit 100 (1967) used the uppercase letters *B*, *C*, *D*, *E*, *F* and *G* for the values 10 to 15.[13]
- The Monrobot XI (1960) used the letters *S*, *T*, *U*, *V*, *W* and *X* for the values 10 to 15.[13]
- The NEC parametron computer NEAC 1103 (1960) used the letters *D*, *G*, *H*, *J*, *K* (and possibly *V*) for values 10–15.[17]
- The Pacific Data Systems 1020 (1964) used the letters *L*, *C*, *A*, *S*, *M* and *D* for the values 10 to 15.[13]
- New numeric symbols and names were introduced in the Bibi-binary notation by Boby Lapointe in 1968. This notation did not become very popular.
- Bruce Alan Martin of Brookhaven National Laboratory considered the choice of A–F "ridiculous". In a 1968 letter to the editor of the CACM, he proposed an entirely new set of symbols based on the bit locations, which did not gain much acceptance.[12]
- Soviet programmable calculators Б3-34 (1980) and similar used the symbols "−", "L", "C", "Г", "E", " " (space) for the values 10 to 15 on their displays.
- Seven-segment display decoder chips used various schemes for outputting values above nine. The Texas Instruments 7446/7447/7448/7449 and 74246/74247/74248/74249 use truncated versions of "2", "3", "4", "5" and "6" for the values 10 to 14. Value 15 (1111 binary) was blank.[18]

Verbal and digital representations

There are no traditional numerals to represent the quantities from ten to fifteen – letters are used as a substitute – and most European languages lack non-decimal names for the numerals above ten. Even though English has names for several non-decimal powers (*pair* for the first binary power, *score* for the first vigesimal power, *dozen*, *gross* and *great gross* for the first three duodecimal powers), no English name describes the hexadecimal powers (decimal 16, 256, 4096, 65536, ...). Some people read hexadecimal numbers digit by digit like a phone number, or using the NATO phonetic alphabet, the Joint Army/Navy Phonetic Alphabet, or a similar ad hoc system. In the wake of the adoption of hexadecimal among IBM System/360 programmers, Robert A. Magnuson suggested in 1968 in Datamation Magazine a pronunciation guide that gave short names to the letters of hexadecimal - for instance, "A" was pronounced "ann", B "bet", C "chris", etc.[19] Another naming system was invented independently by Tim Babb in 2015.[20] An additional naming system has been published online by S. R. Rogers in 2007[21] that tries to make the verbal representation distiguishable in any case, even when the actual number does not contain numbers A-F. Examples are listed in the tables below.

Systems of counting on digits have been devised for both binary and hexadecimal. Arthur C. Clarke suggested using each finger as an on/off bit, allowing finger counting from zero to 1023_{10} on ten fingers.[22] Another system for counting up to FF_{16} (255_{10}) is illustrated on the right.



Hexadecimal finger-counting scheme

Magnusson naming method (1968)

Number	Pronunciation
A	ann
B	bet
C	chris
D	dot
E	ernest
F	frost
1A	annteen
A0	annty
5B	fifty-bet
A01C	annty christeen
1AD0	annteen dotty
3A7D	thirty-ann seventy-dot

Rogers naming method (2007)

Number	Pronunciation
C	twelve
F	fim
11	oneteek
1F	fimteek
50	fiftek
C0	twelftek
100	hundrek
1000	thousek
3E	thirtek-eptwin
E1	eptek-one
C4A	twelve-hundrek-fourtek-ten
1743	one-thousek-seven-hundrek-fourtek-three

Babb naming method (2015)

Number	Pronunciation
1A	abteen
A1	atta-one
1B	bibteen
B1	bibbity-one
1C	cleventeen
C1	city-one
1D	dibbleteen
D1	dickety-one
1E	eggteen
E1	ebbity-one
1F	fleventeen
F1	fleventy-one

Signs

The hexadecimal system can express negative numbers the same way as in decimal: −2A to represent -42_{10} and so on.

Hexadecimal can also be used to express the exact bit patterns used in the processor, so a sequence of hexadecimal digits may represent a signed or even a floating point value. This way, the negative number -42_{10} can be written as FFFF FFD6 in a 32-bit CPU register (in two's-complement), as C228 0000 in a 32-bit FPU register or C045 0000 0000 0000 in a 64-bit FPU register (in the IEEE floating-point standard).

Hexadecimal exponential notation

Just as decimal numbers can be represented in exponential notation, so too can hexadecimal numbers. By convention, the letter *P* (or *p*, for "power") represents *times two raised to the power of*, whereas *E* (or *e*) serves a similar purpose in decimal as part of the E notation. The number after the *P* is *decimal* and represents the *binary* exponent.

Usually the number is normalised so that the leading hexadecimal digit is 1 (unless the value is exactly 0).

Example: 1.3DEp42 represents $1.3\text{DE}_{16} \times 2^{42}$.

Hexadecimal exponential notation is required by the IEEE 754-2008 binary floating-point standard. This notation can be used for floating-point literals in the C99 edition of the C programming language.[23] Using the *%a* or *%A* conversion specifiers, this notation can be produced by implementations of the *printf* family of functions following the C99 specification[24] and Single Unix Specification (IEEE Std 1003.1) POSIX standard.[25]

Conversion

Binary conversion

Most computers manipulate binary data, but it is difficult for humans to work with the large number of digits

for even a relatively small binary number. Although most humans are familiar with the base 10 system, it is much easier to map binary to hexadecimal than to decimal because each hexadecimal digit maps to a whole number of bits (4_{10}). This example converts 1111_2 to base ten. Since each position in a binary numeral can contain either a 1 or a 0, its value may be easily determined by its position from the right:

- $0001_2 = 1_{10}$
- $0010_2 = 2_{10}$
- $0100_2 = 4_{10}$
- $1000_2 = 8_{10}$

Therefore:

$$\begin{aligned} 1111_2 &= 8_{10} + 4_{10} + 2_{10} + 1_{10} \\ &= 15_{10} \end{aligned}$$

With little practice, mapping 1111_2 to F_{16} in one step becomes easy: see table in written representation. The advantage of using hexadecimal rather than decimal increases rapidly with the size of the number. When the number becomes large, conversion to decimal is very tedious. However, when mapping to hexadecimal, it is trivial to regard the binary string as 4-digit groups and map each to a single hexadecimal digit.

This example shows the conversion of a binary number to decimal, mapping each digit to the decimal value, and adding the results.

$$\begin{aligned} (01011110101101010010)_2 &= 262144_{10} + 65536_{10} + 32768_{10} + 16384_{10} + 8192_{10} + \\ &\quad 2048_{10} + 512_{10} + 256_{10} + 64_{10} + 16_{10} + 2_{10} \\ &= 387922_{10} \end{aligned}$$

Compare this to the conversion to hexadecimal, where each group of four digits can be considered independently, and converted directly:

$$\begin{aligned} (01011110101101010010)_2 &= 0101\ 1110\ 1011\ 0101\ 0010_2 \\ &= \quad 5 \quad\ E \quad\ B \quad\ 5 \quad\ 2_{16} \\ &= 5EB52_{16} \end{aligned}$$

The conversion from hexadecimal to binary is equally direct.

Other simple conversions

Although quaternary (base 4) is little used, it can easily be converted to and from hexadecimal or binary. Each hexadecimal digit corresponds to a pair of quaternary digits and each quaternary digit corresponds to a pair of binary digits. In the above example 5 E B 5 2_{16} = 11 32 23 11 02_4.

The octal (base 8) system can also be converted with relative ease, although not quite as trivially as with bases 2 and 4. Each octal digit corresponds to three binary digits, rather than four. Therefore we can convert between octal and hexadecimal via an intermediate conversion to binary followed by regrouping the binary digits in groups of either three or four.

Division-remainder in source base

As with all bases there is a simple algorithm for converting a representation of a number to hexadecimal by doing integer division and remainder operations in the source base. In theory, this is possible from any base, but for most humans only decimal and for most computers only binary (which can be converted by far more efficient methods) can be easily handled with this method.

Let d be the number to represent in hexadecimal, and the series $h_i h_{i-1} ... h_2 h_1$ be the hexadecimal digits representing the number.

1. i ← 1

2. $h_i \leftarrow d \bmod 16$
3. $d \leftarrow (d - h_i) / 16$
4. If d = 0 (return series h_i) else increment i and go to step 2

"16" may be replaced with any other base that may be desired.

The following is a JavaScript implementation of the above algorithm for converting any number to a hexadecimal in String representation. Its purpose is to illustrate the above algorithm. To work with data seriously, however, it is much more advisable to work with bitwise operators.

```
function toHex(d) {
  var r = d % 16;
  if (d - r == 0) {
    return toChar(r);
  }
  return toHex( (d - r)/16 ) + toChar(r);
}

function toChar(n) {
  const alpha = "0123456789ABCDEF";
  return alpha.charAt(n);
}
```

Conversion through addition and multiplication

It is also possible to make the conversion by assigning each place in the source base the hexadecimal representation of its place value — before carrying out multiplication and addition to get the final representation. For example, to convert the number B3AD to decimal, one can split the hexadecimal number into its digits: B (11_{10}), 3 (3_{10}), A (10_{10}) and D (13_{10}), and then get the final result by multiplying each decimal representation by 16^p (p being the corresponding hex digit position, counting from right to left, beginning with 0). In this case, we have that:

$$\mathrm{B3AD} = (11 \times 16^3) + (3 \times 16^2) + (10 \times 16^1) + (13 \times 16^0)$$

which is 45997 in base 10.

×	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
1	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
2	2	4	6	8	A	C	E	10	12	14	16	18	1A	1C	1E	20
3	3	6	9	C	F	12	15	18	1B	1E	21	24	27	2A	2D	30
4	4	8	C	10	14	18	1C	20	24	28	2C	30	34	38	3C	40
5	5	A	F	14	19	1E	23	28	2D	32	37	3C	41	46	4B	50
6	6	C	12	18	1E	24	2A	30	36	3C	42	48	4E	54	5A	60
7	7	E	15	1C	23	2A	31	38	3F	46	4D	54	5B	62	69	70
8	8	10	18	20	28	30	38	40	48	50	58	60	68	70	78	80
9	9	12	1B	24	2D	36	3F	48	51	5A	63	6C	75	7E	87	90
A	A	14	1E	28	32	3C	46	50	5A	64	6E	78	82	8C	96	A0
B	B	16	21	2C	37	42	4D	58	63	6E	79	84	8F	9A	A5	B0
C	C	18	24	30	3C	48	54	60	6C	78	84	90	9C	A8	B4	C0
D	D	1A	27	34	41	4E	5B	68	75	82	8F	9C	A9	B6	C3	D0
E	E	1C	2A	38	46	54	62	70	7E	8C	9A	A8	B6	C4	D2	E0
F	F	1E	2D	3C	4B	5A	69	78	87	96	A5	B4	C3	D2	E1	F0
10	10	20	30	40	50	60	70	80	90	A0	B0	C0	D0	E0	F0	100

A hexadecimal multiplication table

Tools for conversion

Most modern computer systems with graphical user interfaces provide a built-in calculator utility capable of performing conversions between the various radices, and in most cases would include the hexadecimal as well.

In Microsoft Windows, the Calculator utility can be set to Scientific mode (called Programmer mode in some versions), which allows conversions between radix 16 (hexadecimal), 10 (decimal), 8 (octal) and 2 (binary), the bases most commonly used by programmers. In Scientific Mode, the on-screen numeric keypad includes the hexadecimal digits A through F, which are active when "Hex" is selected. In hex mode, however, the Windows Calculator supports only integers.

Elementary arithmetic

Elementary operations such additions, subtractions, multiplications and divisions can be carried out indirectly through conversion to an alternate numeral system, such as the decimal system, since it's the most commonly adopted system, or the binary system, since each hex digit corresponds to four binary digits,

Alternatively, one can also perform elementary operations directly within the hex system itself — by relying on its addition/multiplication tables and its corresponding standard algorithms such as long division and the traditional subtraction algorithm.[26]

Real numbers

Rational numbers

As with other numeral systems, the hexadecimal system can be used to represent rational numbers, although repeating expansions are common since sixteen (10_{16}) has only a single prime factor; two.

For any base, 0.1 (or "1/10") is always equivalent to one divided by the representation of that base value in its own number system. Thus, whether dividing one by two for binary or dividing one by sixteen for hexadecimal, both of these fractions are written as `0.1`. Because the radix 16 is a perfect square (4^2), fractions expressed in hexadecimal have an odd period much more often than decimal ones, and there are no cyclic numbers (other than trivial single digits). Recurring digits are exhibited when the denominator in lowest terms has a prime factor not found in the radix; thus, when using hexadecimal notation, all fractions with denominators that are not a power of two result in an infinite string of recurring digits (such as thirds and fifths). This makes hexadecimal (and binary) less convenient than decimal for representing rational numbers since a larger proportion lie outside its range of finite representation.

All rational numbers finitely representable in hexadecimal are also finitely representable in decimal, duodecimal and sexagesimal: that is, any hexadecimal number with a finite number of digits also has a finite number of digits when expressed in those other bases. Conversely, only a fraction of those finitely representable in the latter bases are finitely representable in hexadecimal. For example, decimal 0.1 corresponds to the infinite recurring representation $0.1\overline{9}$ in hexadecimal. However, hexadecimal is more efficient than duodecimal and sexagesimal for representing fractions with powers of two in the denominator. For example, 0.0625_{10} (one sixteenth) is equivalent to 0.1_{16}, 0.09_{12}, and $0;3,45_{60}$.

n	Decimal Prime factors of base, b = 10: 2, 5; b − 1 = 9: 3; b + 1 = 11: 11			Hexadecimal Prime factors of base, b = 16_{10} = 10: 2; b − 1 = 15_{10} = F: 3, 5; b + 1 = 17_{10} = 11: 11		
	Fraction	Prime factors	Positional representation	Positional representation	Prime factors	Fraction(1/n)
2	1/2	2	**0.5**	**0.8**	2	1/2
3	1/3	3	**0.**3333... = **0.**$\overline{3}$	**0.**5555... = **0.**$\overline{5}$	3	1/3
4	1/4	2	**0.25**	**0.4**	2	1/4
5	1/5	5	**0.2**	**0.**$\overline{3}$	5	1/5
6	1/6	2, 3	**0.1**$\overline{6}$	**0.2**$\overline{A}$	2, 3	1/6
7	1/7	7	**0.**$\overline{142857}$	**0.**$\overline{249}$	7	1/7
8	1/8	2	**0.125**	**0.2**	2	1/8
9	1/9	3	**0.**$\overline{1}$	**0.**$\overline{1C7}$	3	1/9
10	1/10	2, 5	**0.1**	**0.1**$\overline{9}$	2, 5	1/A
11	1/11	11	**0.**$\overline{09}$	**0.**$\overline{1745D}$	B	1/B
12	1/12	2, 3	**0.08**$\overline{3}$	**0.1**$\overline{5}$	2, 3	1/C
13	1/13	13	**0.**$\overline{076923}$	**0.**$\overline{13B}$	D	1/D
14	1/14	2, 7	**0.0**$\overline{714285}$	**0.1**$\overline{249}$	2, 7	1/E
15	1/15	3, 5	**0.0**$\overline{6}$	**0.**$\overline{1}$	3, 5	1/F
16	1/16	2	**0.0625**	**0.1**	2	1/10
17	1/17	17	**0.**$\overline{0588235294117647}$	**0.0**$\overline{F}$	11	1/11
18	1/18	2, 3	**0.0**$\overline{5}$	**0.0**$\overline{E38}$	2, 3	1/12
19	1/19	19	**0.**$\overline{052631578947368421}$	**0.**$\overline{0D79435E5}$	13	1/13
20	1/20	2, 5	**0.05**	**0.0**$\overline{C}$	2, 5	1/14
21	1/21	3, 7	**0.**$\overline{047619}$	**0.**$\overline{0C3}$	3, 7	1/15
22	1/22	2, 11	**0.0**$\overline{45}$	**0.0**$\overline{BA2E8}$	2, B	1/16
23	1/23	23	**0.**$\overline{0434782608695652173913}$	**0.**$\overline{0B21642C859}$	17	1/17
24	1/24	2, 3	**0.041**$\overline{6}$	**0.0**$\overline{A}$	2, 3	1/18
25	1/25	5	**0.04**	**0.**$\overline{0A3D7}$	5	1/19
26	1/26	2, 13	**0.0**$\overline{384615}$	**0.0**$\overline{9D8}$	2, D	1/1A
27	1/27	3	**0.**$\overline{037}$	**0.**$\overline{097B425ED}$	3	1/1B
28	1/28	2, 7	**0.03**$\overline{571428}$	**0.0**$\overline{924}$	2, 7	1/1C
29	1/29	29	**0.**$\overline{0344827586206896551724137931}$	**0.**$\overline{08D3DCB}$	1D	1/1D
30	1/30	2, 3, 5	**0.0**$\overline{3}$	**0.0**$\overline{8}$	2, 3, 5	1/1E
31	1/31	31	**0.**$\overline{032258064516129}$	**0.**$\overline{08421}$	1F	1/1F
32	1/32	2	**0.03125**	**0.08**	2	1/20
33	1/33	3, 11	**0.**$\overline{03}$	**0.**$\overline{07C1F}$	3, B	1/21
34	1/34	2, 17	**0.0**$\overline{2941176470588235}$	**0.0**$\overline{78}$	2, 11	1/22
35	1/35	5, 7	**0.0**$\overline{285714}$	**0.**$\overline{075}$	5, 7	1/23

36	1/36	2, 3	**0.02**$\overline{7}$	**0.0**$\overline{71C}$	2, 3	1/24

Irrational numbers

The table below gives the expansions of some common irrational numbers in decimal and hexadecimal.

Number	Positional representation	
	Decimal	Hexadecimal
$\sqrt{2}$ (the length of the diagonal of a unit square)	1.414 213 562 373 095 048...	1.6A09E667F3BCD...
$\sqrt{3}$ (the length of the diagonal of a unit cube)	1.732 050 807 568 877 293...	1.BB67AE8584CAA...
$\sqrt{5}$ (the length of the diagonal of a 1×2 rectangle)	2.236 067 977 499 789 696...	2.3C6EF372FE95...
φ (phi, the golden ratio = $(1+\sqrt{5})/2$)	1.618 033 988 749 894 848...	1.9E3779B97F4A...
π (pi, the ratio of circumference to diameter of a circle)	3.141 592 653 589 793 238 462 643 383 279 502 884 197 169 399 375 105...	3.243F6A8885A308D313198A2E0 3707344A4093822299F31D008...
e (the base of the natural logarithm)	2.718 281 828 459 045 235...	2.B7E151628AED2A6B...
τ (the Thue–Morse constant)	0.412 454 033 640 107 597...	0.6996 9669 9669 6996...
γ (the limiting difference between the harmonic series and the natural logarithm)	0.577 215 664 901 532 860...	0.93C467E37DB0C7A4D1B...

Powers

Powers of two have very simple expansions in hexadecimal. The first sixteen powers of two are shown below.

2^x	Value	Value (Decimal)
2^0	1	1
2^1	2	2
2^2	4	4
2^3	8	8
2^4	10_{hex}	16_{dec}
2^5	20_{hex}	32_{dec}
2^6	40_{hex}	64_{dec}
2^7	80_{hex}	128_{dec}
2^8	100_{hex}	256_{dec}
2^9	200_{hex}	512_{dec}
2^A ($2^{10_{dec}}$)	400_{hex}	1024_{dec}
2^B ($2^{11_{dec}}$)	800_{hex}	2048_{dec}
2^C ($2^{12_{dec}}$)	1000_{hex}	4096_{dec}
2^D ($2^{13_{dec}}$)	2000_{hex}	8192_{dec}
2^E ($2^{14_{dec}}$)	4000_{hex}	$16{,}384_{dec}$
2^F ($2^{15_{dec}}$)	8000_{hex}	$32{,}768_{dec}$
2^{10} ($2^{16_{dec}}$)	10000_{hex}	$65{,}536_{dec}$

Cultural

Etymology

The word *hexadecimal* is composed of *hexa-*, derived from the Greek ἕξ (hex) for *six*, and *-decimal*, derived from the Latin for *tenth*. Webster's Third New International online derives *hexadecimal* as an alteration of the all-Latin *sexadecimal* (which appears in the earlier Bendix documentation). The earliest date attested for *hexadecimal* in Merriam-Webster Collegiate online is 1954, placing it safely in the category of international scientific vocabulary (ISV). It is common in ISV to mix Greek and Latin combining forms freely. The word *sexagesimal* (for base 60) retains the Latin prefix. Donald Knuth has pointed out that the etymologically correct term is *senidenary* (or possibly, *sedenary*), from the Latin term for *grouped by 16*. (The terms *binary*, *ternary* and *quaternary* are from the same Latin construction, and the etymologically correct terms for *decimal* and *octal* arithmetic are *denary* and *octonary*, respectively.)[27] Alfred B. Taylor used *senidenary* in his mid-1800s work on alternative number bases, although he rejected base 16 because of its "incommodious number of digits".[28][29] Schwartzman notes that the expected form from usual Latin phrasing would be *sexadecimal*, but computer hackers would be tempted to shorten that word to *sex*.[30] The etymologically proper Greek term would be *hexadecadic* / *ἑξαδεκαδικός* / *hexadekadikós* (although in Modern Greek, *decahexadic* / *δεκαεξαδικός* / *dekaexadikos* is more commonly used).

Use in Chinese culture

The traditional Chinese units of measurement were base-16. For example, one jīn (斤) in the old system equals sixteen taels. The suanpan (Chinese abacus) can be used to perform hexadecimal calculations such as additions and subtractions.[31]

Primary numeral system

As with the duodecimal system, there have been occasional attempts to promote hexadecimal as the preferred numeral system. These attempts often propose specific pronunciation and symbols for the individual numerals.[32] Some proposals unify standard measures so that they are multiples of 16.[33][34][35]

An example of unified standard measures is hexadecimal time, which subdivides a day by 16 so that there are 16 "hexhours" in a day.[35]

Base16 (Transfer encoding)

Base16 (as a proper name without a space) can also refer to a binary to text encoding belonging to the same family as Base32, Base58, and Base64.

In this case, data is broken into 4-bit sequences, and each value (between 0 and 15 inclusively) is encoded using 16 symbols from the ASCII character set. Although any 16 symbols from the ASCII character set can be used, in practice the ASCII digits '0'-'9' and the letters 'A'-'F' (or the lowercase 'a'-'f') are always chosen in order to align with standard written notation for hexadecimal numbers.

There are several advantages of Base16 encoding:

- Most programming languages already have facilities to parse ASCII-encoded hexadecimal
- Being exactly half a byte, 4-bits is easier to process than the 5 or 6 bits of Base32 and Base64 respectively
- The symbols 0-9 and A-F are universal in hexadecimal notation, so it is easily understood at a glance without needing to rely on a symbol lookup table
- Many CPU architectures have dedicated instructions that allow access to a half-byte (otherwise known as a "Nibble"), making it more efficient in hardware than Base32 and Base64

The main disadvantages of Base16 encoding are:

- Space efficiency is only 50%, since each 4-bit value from the original data will be encoded as an 8-bit byte. In contrast, Base32 and Base64 encodings have a space efficiency of 63% and 75% respectively.
- Possible added complexity of having to accept both uppercase and lowercase letters

Support for Base16 encoding is ubiquitous in modern computing. It is the basis for the W3C standard for URL Percent Encoding, where a character is replaced with a percent sign "%" and its Base16-encoded form. Most modern programming languages directly include support for formatting and parsing Base16-encoded numbers.

See also

- Base32, Base64 (content encoding schemes)
- Hexadecimal time
- IBM hexadecimal floating point
- Hex editor
- Hex dump
- Bailey–Borwein–Plouffe formula (BBP)
- Hexspeak

References

1. "The Unicode Standard, Version 7" (https://www.unicode.org/charts/PDF/U2000.pdf) (PDF). *Unicode*. Retrieved October 28, 2018.
2. "Hexadecimal web colors explained" (https://web.archive.org/web/20060422004336/http://www.web-colors-explained.com/hex.php). Archived from the original (http://www.web-colors-explained.com/hex.php) on 2006-04-22. Retrieved 2006-01-11.
3. The string `"\x1B[0m\x1B[25;1H"` specifies the character sequence `Esc [ 0 m Esc [ 2 5 ; 1 H Nul`. These are the escape sequences used on an ANSI terminal that reset the character set and color, and then move the cursor to line 25.

4. "ISO-8859-1 (ISO Latin 1) Character Encoding" (https://www.ic.unicamp.br/~stolfi/EXPORT/www/ISO-8859-1-Encoding.html). *www.ic.unicamp.br*. Retrieved 2019-06-26.
5. "Modula-2 - Vocabulary and representation" (http://modula2.org/reference/vocabulary.php). *Modula -2*. Retrieved 1 November 2015.
6. The VHDL MINI-REFERENCE: VHDL IDENTIFIERS, NUMBERS, STRINGS, AND EXPRESSIONS (http://www.eng.auburn.edu/department/ee/mgc/vhdl.html#numbers) Archived (https://web.archive.org/web/20071226170804/http://eng.auburn.edu/department/ee/mgc/vhdl.html#numbers) 2007-12-26 at the Wayback Machine
7. "*read-base* variable in Common Lisp" (http://www.lispworks.com/documentation/HyperSpec/Body/v_rd_bas.htm). *CLHS*.
8. "*print-base* variable in Common Lisp" (http://www.lispworks.com/documentation/HyperSpec/Body/v_pr_bas.htm#STprint-baseST). *CLHS*.
9. MSX is Coming — Part 2: Inside MSX (http://www.atarimagazines.com/compute/issue56/107_1_MSX_IS_COMING.php) Compute!, issue 56, January 1985, p. 52
10. BBC BASIC programs are not fully portable to Microsoft BASIC (without modification) since the latter takes & to prefix octal values. (Microsoft BASIC primarily uses &0 to prefix octal, and it uses &H to prefix hexadecimal, but the ampersand alone yields a default interpretation as an octal prefix.
11. Donald E. Knuth. *The TeXbook* (Computers and Typesetting, Volume A). Reading, Massachusetts: Addison–Wesley, 1984. ISBN 0-201-13448-9. The source code of the book in TeX (http://www.ctan.org/tex-archive/systems/knuth/tex/texbook.tex) Archived (https://web.archive.org/web/20070927224129/http://www.ctan.org/tex-archive/systems/knuth/tex/texbook.tex) 2007-09-27 at the Wayback Machine (and a required set of macros CTAN.org (ftp://tug.ctan.org/pub/tex-archive/systems/knuth/lib/manmac.tex)) is available online on CTAN.
12. Martin, Bruce Alan (October 1968). "Letters to the editor: On binary notation". *Communications of the ACM*. Associated Universities Inc. **11** (10): 658. doi:10.1145/364096.364107 (https://doi.org/10.1145%2F364096.364107).
13. Savard, John J. G. (2018) [2005]. "Computer Arithmetic" (http://www.quadibloc.com/comp/cp02.htm). *quadibloc*. The Early Days of Hexadecimal. Archived (https://web.archive.org/web/20180716102439/http://www.quadibloc.com/comp/cp02.htm) from the original on 2018-07-16. Retrieved 2018-07-16.
14. "2.1.3 Sexadecimal notation". *G15D Programmer's Reference Manual* (http://bitsavers.trailing-edge.com/pdf/bendix/g-15/G15D_Programmers_Ref_Man.pdf) (PDF). Los Angeles, CA, USA: Bendix Computer, Division of Bendix Aviation Corporation. p. 4. Archived (https://web.archive.org/web/20170601222212/http://bitsavers.trailing-edge.com/pdf/bendix/g-15/G15D_Programmers_Ref_Man.pdf) (PDF) from the original on 2017-06-01. Retrieved 2017-06-01. "This base is used because a group of four bits can represent any one of sixteen different numbers (zero to fifteen). By assigning a symbol to each of these combinations we arrive at a notation called sexadecimal (usually hex in conversation because nobody wants to abbreviate sex). The symbols in the sexadecimal language are the ten decimal digits and, on the G-15 typewriter, the letters u, v, w, x, y and z. These are arbitrary markings; other computers may use different alphabet characters for these last six digits."
15. Gill, S.; Neagher, R. E.; Muller, D. E.; Nash, J. P.; Robertson, J. E.; Shapin, T.; Whesler, D. J. (1956-09-01). Nash, J. P. (ed.). "ILLIAC Programming - A Guide to the Preparation of Problems For Solution by the University of Illinois Digital Computer" (http://www.textfiles.com/bitsavers/pdf/illiac/ILLIAC/ILLIAC_programming_Sep56.pdf) (PDF). *bitsavers.org* (Fourth printing. Revised and corrected ed.). Urbana, Illinois, USA: Digital Computer Laboratory, Graduate College, University of Illinois. pp. 3–2. Archived (https://web.archive.org/web/20170531153804/http://www.textfiles.com/bitsavers/pdf/illiac/ILLIAC/ILLIAC_programming_Sep56.pdf) (PDF) from the original on 2017-05-31. Retrieved 2014-12-18.
16. *ROYAL PRECISION Electronic Computer LGP - 30 PROGRAMMING MANUAL* (http://ed-thelen.org/comp-hist/lgp-30-man.html#R4.13). Port Chester, New York: Royal McBee Corporation. April 1957. Archived (https://web.archive.org/web/20170531153004/http://ed-thelen.org/comp-hist/lgp-30-man.html) from the original on 2017-05-31. Retrieved 2017-05-31. (NB. This somewhat odd sequence was from the next six sequential numeric keyboard codes in the LGP-30's 6-bit character code.)
17. *NEC Parametron Digital Computer Type NEAC-1103* (http://archive.computerhistory.org/resources/text/NEC/NEC.1103.1958102646285.pdf) (PDF). Tokyo, Japan: Nippon Electric Company Ltd. 1960. Cat. No. 3405-C. Archived (https://web.archive.org/web/20170531112850/http://archive.computerhistory.org/resources/text/NEC/NEC.1103.1958102646285.pdf) (PDF) from the original on 2017-05-31. Retrieved 2017-05-31.

18. *BCD-to-Seven-Segment Decoders/Drivers: SN54246/SN54247/SN54LS247, SN54LS248 SN74246/SN74247 /SN74LS247/SN74LS248* (http://www.ralphselectronics.com/ProductImages/SEMI-SN74247N.PDF) (PDF), Texas Instruments, March 1988 [March 1974], SDLS083, archived (https://web.archive.org/web/20170329223343/http://www.ralphselectronics.com/ProductImages/SEMI-SN74247N.PDF) (PDF) from the original on 2017-03-29, retrieved 2017-03-30, "[…] They can be used interchangeable in present or future designs to offer designers a choice between two indicator fonts. The '46A, '47A, 'LS47, and 'LS48 compose the 6 and the 9 without tails and the '246, '247, 'LS247, and 'LS248 compose the 6 and the 0 with tails. Composition of all other characters, including display patterns for BCD inputs above nine, is identical. […] Display patterns for BCD input counts above 9 are unique symbols to authenticate input conditions. […]"
19. Magnuson, Robert (January 1968). "A Hexadecimal Pronunciation Guide". *Datamation*. **14** (1): 45.
20. "How to pronounce hexadecimal" (https://www.bzarg.com/p/how-to-pronounce-hexadecimal/). *Bzarg*. Retrieved 2019-08-26.
21. "Hexadecimal Number Words" (http://www.intuitor.com/hex/words.html). *Intuitor*. Retrieved 2019-08-26.
22. Clarke, Arthur; Pohl, Frederik (2008). *The Last Theorem*. Ballantine. p. 91. ISBN 978-0007289981.
23. "ISO/IEC 9899:1999 - Programming languages - C" (http://www.iso.org/iso/iso_catalogue/catalogue_ics/catalogue_detail_ics.htm?csnumber=29237). *ISO*. Iso.org. 2011-12-08. Retrieved 2014-04-08.
24. "Rationale for International Standard - Programming Languages - C" (http://www.open-std.org/jtc1/sc22/wg14/www/C99RationaleV5.10.pdf) (PDF). *Open Standards*. 5.10. April 2003. pp. 52, 153–154, 159. Archived (https://web.archive.org/web/20160606072228/http://www.open-std.org/jtc1/sc22/wg14/www/C99RationaleV5.10.pdf) (PDF) from the original on 2016-06-06. Retrieved 2010-10-17.
25. The IEEE and The Open Group (2013) [2001]. "dprintf, fprintf, printf, snprintf, sprintf - print formatted output" (http://pubs.opengroup.org/onlinepubs/9699919799/functions/printf.html). *The Open Group Base Specifications* (Issue 7, IEEE Std 1003.1, 2013 ed.). Archived (https://web.archive.org/web/20160621211105/http://pubs.opengroup.org/onlinepubs/9699919799/functions/printf.html) from the original on 2016-06-21. Retrieved 2016-06-21.
26. "The Definitive Higher Math Guide to Long Division and Its Variants — for Integers" (https://mathvault.ca/long-division/). *Math Vault*. 2019-02-24. Retrieved 2019-06-26.
27. Knuth, Donald. (1969). *The Art of Computer Programming, Volume 2*. ISBN 0-201-03802-1. (Chapter 17.)
28. Alfred B. Taylor, Report on Weights and Measures (https://books.google.com/books?id=X7wLAAAAYAAJ&pg=PP5), Pharmaceutical Association, 8th Annual Session, Boston, Sept. 15, 1859. See pages and 33 and 41.
29. Alfred B. Taylor, "Octonary numeration and its application to a system of weights and measures", *Proc Amer. Phil. Soc.* Vol XXIV (https://books.google.com/books?id=KsAUAAAAYAAJ&pg=PA296), Philadelphia, 1887; pages 296-366. See pages 317 and 322.
30. Schwartzman, S. (1994). *The Words of Mathematics: an etymological dictionary of mathematical terms used in English*. ISBN 0-88385-511-9.
31. "算盤 Hexadecimal Addition & Subtraction on an Chinese Abacus" (http://totton.idirect.com/soroban/Hex_as/). *totton.idirect.com*. Retrieved 2019-06-26.
32. "Base 4^2 Hexadecimal Symbol Proposal" (http://www.hauptmech.com/base42). *Hauptmech*.
33. "Intuitor Hex Headquarters" (http://www.intuitor.com/hex/). *Intuitor*. Retrieved October 28, 2018.
34. Niemietz, Ricardo Cancho (October 21, 2003). "A proposal for addition of the six Hexadecimal digits (A-F) to Unicode" (http://std.dkuug.dk/jtc1/sc2/wg2/docs/n2677). *DKUUG Standardizing*. Retrieved October 28, 2018.
35. Nystrom, John William (1862). *Project of a New System of Arithmetic, Weight, Measure and Coins: Proposed to be called the Tonal System, with Sixteen to the Base* (https://books.google.com/books?id=aNYGAAAAYAAJ). Philadelphia: Lippincott.
